$\Psi(M)$ $d\Psi$

m

$m\,lim$

$C(M)$ dx

$X(M)dx$

M

$L\ M$

Luminosity Functions from Photometric Redshifts I: Techniques

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ABSTRACT

The determination of the galaxy luminosity function is an active and fundamental field in observational cosmology. In this paper we propose a cost effective way of measuring galaxy luminosity functions at faint magnitudes. Our technique employs the use of galaxy redshifts estimated from their multicolor photometry (Connolly et al. 1995). Associated with the redshift estimate is a well defined error distribution. We have derived a variant of Lynden–Bell's (1971) C–method that considers, for each galaxy, the probability distribution in absolute magnitude resultant from the redshift error. This technique is tested through simulations and potential biases are quantified. We then apply the technique to a sample of galaxies with multicolor photometric data at moderately faint ($B \approx 23$) limits, and compare the results to a subset of these data with spectroscopic redshifts. We find that the luminosity function derived from the photometric redshifts is consistent with that determined from spectroscopic redshifts.

1. Introduction

The determination of the galaxy luminosity function is of crucial importance to cosmology. Knowledge of the local luminosity function is necessary to interpret the results of faint galaxy counts and their implications for cosmography and galaxy evolution. A more explicit measure of galaxy evolution is the variation of the luminosity function with redshift and spectral type. Considering that the luminosity functions may vary with redshift, color, local density, morphology, spectral type, size, etc., many redshifts are required to get a detailed picture of the multivariate galaxy distribution.

The difficulty in determining the luminosity function from a magnitude limited sample of galaxies is that intrinsically bright galaxies can be seen out to great distances while intrinsically faint galaxies can be seen only relatively nearby. One way to account for this observational bias is to weight each galaxy's contribution by the inverse of the volume over which it could have been observed. This is Schmidt's (1968) $1/V_{max}$ technique. The disadvantage of this approach is that it does not properly account for galaxy clustering. Nevertheless, it is still widely used today, (e.g. Lilly et al. 1995). There are several non–parametric techniques which can account for galaxy clustering: the method of Choloniewski (1986), the stepwise maximum–likelihood method of Efstathiou, Ellis and Peterson (1988), and the C-method on which our work is based (Lynden–Bell 1971).

Developments in the experimental determination of the luminosity function have been tied to large redshift surveys. Here we will outline a few of the major contributions. The Stromlo-APM redshift survey (Loveday et al. 1992) contained 1769 galaxies limited at $b_J = 18.7$ ($B \approx b_J + 0.3$) and found a local luminosity function which was well fitted by a Schecter function (Schecter 1976), with parameters $\phi^* = 1.75 \times 10^{-3} h_{50}^{-3}$, $M_{B_J}^* = -21.0 + 5\log h_{50}$ and $\alpha = -1.0$. The luminosity function from the CfA redshift survey (deLapparent, Geller and Huchra 1989), later extended in

Marzke et al. (1994), consists of 9063 galaxies at a limit of $m_z = 17.0$. Their luminosity function had a much higher normalization $\phi^* = 5.0 \times 10^{-3} h_{50}^{-3}$ and three times as many faint galaxies as predicted by the extrapolation of a flat ($\alpha = -1.0$) luminosity function. In contrast to these relatively local samples, the Canada-France redshift survey (Lilly et al. 1995, henceforth CFRS) is much deeper, containing 591 galaxies with a median redshift of $\langle z \rangle = 0.56$. This landmark sample of galaxies allows them to explicitly measure the evolution of the luminosity function with redshift and even to separate the sample by color.

In this paper we describe a new technique which will allow the determination of the luminosity function from deep samples with just as many galaxies as the CFRS, but with far less observational cost. The use of the photometric redshift technique of Connolly et al. (1995), hereafter C95, provides 300 times as efficient use of telescope time as spectroscopic techniques. The tradeoff for the increased efficiency is an error in the redshift determinations. Our technique accounts for this error by considering each galaxy as having a distribution in redshift. This distribution in redshift, folded in with the K–corrections, leads to a distribution in absolute magnitude. As pointed out by Efstathiou, Ellis and Peterson (1988) one shortcoming of the standard non–parametric techniques for determining the luminosity function is that they take no account of our expectation that the luminosity function is smooth. Our approach forces a degree of smoothness onto our luminosity function, and in some sense the information lost due to the redshift errors is partially compensated for by this smoothness assumption.

There are inherent difficulties in attempting to measure a continuous distribution such as the luminosity function from discrete galaxies. The luminosity function techniques deal with this difficulty either by binning the data, or in the case of the C-method by producing a cumulative luminosity function with discrete jumps at the location of each galaxy. When the data are represented by continuous functions such difficulties are naturally averted. The

C-method is easily adapted to handle the continuous data, and the calculation is actually simplified.

Throughout this paper we adopt the values $H_0 = 50\ km\ s^{-1}\ Mpc^{-1}$ and $q_0 = 0.5$. Apparent and absolute magnitudes are denoted by lower and upper case respectively.

2. The Technique

2.1. Photometric redshifts

Photometric redshifts were first proposed as an efficient means of deriving statistical measures of galaxy distances by Baum (1962). Early attempts to derive accurate estimates of galaxy redshifts relied on fitting assumed galaxy evolution models to the observed colors of galaxies (Koo 1985). This resulted in uncertainties in the estimated redshifts in excess of 0.1 (for redshifts greater than 0.35) and non–Gaussian error distributions. More recently C95 have shown that by utilizing the flux information and deriving empirical photometric-redshift relations from existing deep redshift surveys, redshifts could be estimated with a dispersion of $\sigma_z < 0.05$, out to $z = 0.6$.

For this paper we assume that the error distribution in the estimated photometric redshifts can be approximated by a Gaussian, consistent with the findings of C95. From the data of Munn et al. (1996) we find the dispersion in the estimated redshift, as a function of r_F magnitude, to be well described by a linear relation,

$$\sigma_z = 0.051 + 0.0083(r_F - 20). \tag{1}$$

We use this relation to assign an error estimate for each galaxy both in our simulations and in our application to observational data (see 3.1). We note that simulations used to determine the intrinsic dispersion of the photometric-redshift relation have shown that the

current estimate is dominated by the photometric uncertainties in the Munn et al. (1996) data set. To determine the applicability of our technique to future photometric samples, we additionally simulate the effect of reducing the dispersion in the photometric-redshift relation by a factor of three (see C95).

2.2. The 'discrete' C-method

The C-method (c.f. Lynden–Bell 1971, Petrosian 1992) is a powerful estimator for the luminosity function. It is a non-parametric technique which is insensitive to density inhomogeneities and makes full use of all the data. The C-method estimates the cumulative luminosity function which then needs to be differenced in order to give the usual differential form. In this subsection we will describe the C-method and in the next we will describe how we have extended it for use on a photometric redshift data set.

We want to determine the cumulative luminosity function, $\Psi(M_0)$, which is the density of galaxies whose absolute magnitude, M is brighter than, M_0 ($M < M_0$). We actually measure the cumulative distribution $X(M_0)$ which is $\Psi(M_0)$ subject to a set of observational constraints (e.g. apparent magnitude, surface brightness limits). In general,

$$\frac{d\Psi}{\Psi} > \frac{dX}{X}, \tag{2}$$

since it is generally harder to observe fainter galaxies. We therefore wish to construct $C(M)$, a subset of $X(M)$, for which the following relationship holds:

$$\frac{d\Psi}{\Psi} = \frac{dX}{C}. \tag{3}$$

To do this, we need to take the observational constraint on dX and apply that uniformly over X. Namely, $C(M_0)$ is the number of galaxies brighter than M_0 which could have been observed if their absolute magnitude were M_0. The quantities $C(M)$, $X(M)$, and $\Psi(M)$ are

illustrated in Fig 1. To solve for $\Psi(M)$, we integrate equation (3) i.e.,

$$\Psi(M) = A \exp \left[\int_{-\infty}^{M} \frac{dX}{C} \right]. \tag{4}$$

The quantity of interest, the differential luminosity function, is therefore,

$$\Phi(M) = A \exp \left[\int_{-\infty}^{M} \frac{dX}{C} \right] \frac{dX(M)}{C(M)}. \tag{5}$$

When errors are not included $dX(M)$ is a series of Dirac delta functions $dX(M) = \sum_i \delta(M - M_i)$. At the points $M = M_i$ the function $C(M)$ is indeterminate. Because of this, the determination of the integral in Eq. (4) requires further analysis. By carefully considering the integral around these points, Lynden–Bell arrives at the result

$$\Psi(M) = A \exp \prod_i \left(\frac{1 + C(M_i)}{C(M_i)} \right). \tag{6}$$

2.3. Our modified, 'continuous' C-method

In a photometric redshift survey we have a less accurate measure of the galaxy's redshift. The error distribution in redshift, folded in with K-corrections, leads to a probability distribution in absolute magnitude for each galaxy. The function $dX(M)$ is now represented as a smooth function rather than a series of delta functions, and the integral in Eqns (4) and (5) is easily calculated. Each galaxy is represented as a Gaussian distribution in redshift with mean z_i and dispersion σ_i. The photometric redshift procedure is considered as a Gaussian random process, and by integrating over these distributions we recover the "ensemble averaged" luminosity function. The distribution in absolute magnitude includes the effects of K–corrections and consequentially has a much more complicated form. For this reason, when calculating the functions $C(M)$ and $X(M)$, we prefer to work in redshift space. The following analysis is for a complete magnitude limited sample.

Consider the function $z^*(m_i, M)$, which is the redshift at which the ith galaxy will have apparent magnitude m_i and absolute magnitude M. It is the solution to the equation

$$m_i - M = 5 \log_{10}(d_l(z^*)) + 25 + K_i(z^*), \tag{7}$$

where d_l is the luminosity distance in Mpc, and $K_i(z^*)$ is the K-correction for the ith galaxy at redshift z^*. The function $X(M)$ will include the fraction of the galaxy with $z > z^*(m_i, M)$. For the case of a Gaussian error distribution this becomes

$$X(M) = 0.5 \sum_i \mathrm{erfc}\left(\frac{z^*(m_i, M) - z_i}{\sigma_i}\right), \tag{8}$$

where erfc(x) denotes the complimentary error function. A galaxy with absolute magnitude M can only be seen at redshifts less than $z^*(m_{lim}, M)$. The function $C(M)$ will include the fraction of each galaxy between $z^*(m_i, M)$ and $z^*(m_{lim}, M)$.

$$C(M) = 0.5 \sum_i \left[\mathrm{erfc}\left(\frac{z^*(m_i, M) - z_i}{\sigma_i}\right) - \mathrm{erfc}\left(\frac{z^*(m_{lim}, M) - z_i}{\sigma_i}\right)\right]. \tag{9}$$

The relevant quantities are illustrated in Fig 2. Once we have the two functions $C(M)$ and $X(M)$ the calculation of the luminosity function from equation (4) is straightforward.

One of the problems in constructing the galaxy luminosity functions is that it is not necessarily possible to solve for both the intrinsic luminosity distribution and the variations of the density with redshift. Using a technique such as $1/V_{max}$ (Schmidt 1968), which makes the assumption of uniform density, the only requirement is that the data span the range in absolute magnitude over which we are estimating the luminosity function. Since we wish to make no assumptions regarding the density fluctuations, we must make the further requirement that the data be strongly connected. Consider a situation where there are two groups of galaxies, one near and one far. The galaxies in the near group are all too faint to be seen if they were transported to the far group. In this situation there is no way to determine the relative density fluctuations between the near and far groups. Hence it

is impossible to construct the full luminosity function. Such a situation is indicated when $C(M) = 0$. When this happens, the luminosity function is split into two halves, brighter and fainter than M, which cannot be normalized with respect to one another. With our technique, where each galaxy occupies a range in absolute magnitudes, the chances that $C = 0$ for a real magnitude limited sample of galaxies are greatly reduced.

2.4. Normalization of the luminosity function

The procedure described above produces an unnormalized estimate of the luminosity function. We normalize by calculating the expected number of galaxies in a certain range in absolute magnitude and redshift and comparing that to the number observed. In order to do this calculation we need to explicitly calculate the variation of galaxy density with redshift. This calculation can be done exactly like that of the luminosity function, simply by replacing absolute magnitude, M, with redshift, z (see Eqns 4 - 9). We now have unnormalized estimates of the luminosity function, $\Phi(M)$, and the galaxy density as a function of redshift, $\rho(z)dV/dz$. The normalized luminosity function will be denoted by $\hat{\Phi}$. To normalize the luminosity function we need the quantity ρ_0, the integral of the luminosity function. We need to choose a range of redshift (z_0, z_1) over which to calculate ρ_0:

$$\rho_0 = \frac{\int\limits_{z0}^{z1} \frac{\rho(z)dV}{dz}dz}{\int\limits_{z0}^{z1} \frac{dV}{dz}dz}.$$ (10)

We can now normalize by integrating the unnormalized luminosity function and the relative density fluctuations, $\rho(z)/\rho_0$ to get the expected number of galaxies. Comparing this number with the observed number of galaxies will give us the normalization. However, to do this integral we need to know the intrinsic distribution of galaxy types and their K–corrections. Therefore we must limit the integral to a range of absolute magnitude and redshift where the entire spectrum of galaxy types can be observed. We can now calculate

the number of galaxies which we expect to observe in the range $M0 < M < M1$ and $z0 < z < z1$.

$$N_e = \sum_{i=1}^{4} f_i \int\limits_{M0}^{M1} \int\limits_{z0}^{z1} A\Phi(M)\frac{\rho(z)}{\rho_0}\frac{dV}{dz}\Theta(z^*(m_{lim}, M) - z)dzdM, \tag{11}$$

where f_i represents the fraction of galaxies assigned to the ith spectral type based on their $b_J - r_F$ colors, and $\Theta(x)$ is a step function. The number of galaxies observed between the redshift and absolute magnitude limits is N_o. Since the galaxies are represented as a distribution both in redshift and absolute magnitude this is not a integer quantity. Comparing these quantities gives us the normalization,

$$\hat{\Phi}(M) = \Phi(M)\frac{N_o}{N_e} \tag{12}$$

2.5.   Errors and Biases

Errors are estimated using bootstrap techniques. Artificial samples are generated by randomly picking galaxies from the real sample. Each artificial sample has the same number of galaxies as the original sample. Individual galaxies may be picked multiple times or not at all. 100 samples are generated in this manner and the luminosity function is calculated for each. The variance in the luminosity function is then given by the variance of the artificial samples. We use the random number generator `ran1` from Numerical Recipes (Press et al. 1986).

It is well known that the C-method is an unbiased estimator of the luminosity function. However, when we are dealing with fuzzy data this will no longer be the case. The broad distribution in absolute magnitude acts as a smoothing kernel, preferentially scattering galaxies away from M^*, where the distribution is peaked, and towards the bright and faint ends of the distribution. This produces a bias analogous to Eddington or Malmquist bias (Eddington 1940, Malmquist 1920).

We have attempted a correction for this bias. Since the absolute magnitude distribution is different for each galaxy depending on its redshift, redshift error and spectral type; the correction is best handled by Monte-Carlo techniques. The problem is as follows: given a measured absolute magnitude distribution $\left(\frac{dX(M)}{dM}\right)_m$, what is the most likely true distribution $\left(\frac{dX(M)}{dM}\right)_t$. Once this is known the luminosity function can be corrected using,

$$\hat{\Phi}(M) = \Phi(M) \left(\frac{dX(M)}{dM}\right)_m^{-1} \left(\frac{dX(M)}{dM}\right)_t. \tag{13}$$

We can estimate the effect of going from the true to the measured distribution by randomly scattering each galaxy in redshift according to its error distribution. We invert this procedure in an iterative manner. The noisiness of the sample prevents absolute convergence so we have terminated the procedure after two iterations. We have performed simulations to estimate the effect of both the bias and its correction.

Simulated galaxies are placed within the simulated volume according to a given Schecter function. Galaxy types are randomly assigned to one of the four templates from Kinney et al. (1996) (1996), as described in section 3.2. The apparent magnitude is then calculated using the true redshift and the galaxy's K-correction. The galaxy is either included or excluded from the sample according to its apparent magnitude. The galaxy's $b_J - r_F$ color is determined from the template which is used to assign the error in redshift as in Eqn 1. The estimated redshift is then scattered from the true redshift by an amount drawn from its Gaussian error distribution.

We present the results from four simulation scenarios with 100 realizations each. The number of galaxies in the simulations is allowed to vary with each realization. The normalization of the input luminosity function is chosen so that there are roughly 800 galaxies in each of the simulations. We vary both the input luminosity function as well as the error distribution. As discussed in C95, the errors of Eqn 1 are dominated by errors in the photographic photometry. With better photometry galaxy redshifts could be

estimated three times as accurately. Simulations are performed for both the present as well as potential errors on both steep and flat luminosity functions ($\alpha = -1.5$ and $\alpha = -1.0$).

Fig 3 shows the deformation vectors for the recovered Schecter parameters for the simulations. In all cases the slope is overestimated, although our correction does seem to reduce the effect. We slightly over–correct the bright end, systematically overestimating M^* by 0.2 magnitudes. Fig 4 shows the recovered luminosity functions with the input Schecter luminosity function. Steep luminosity functions are reproduced much better than flat ones. This is easily understood as there are many more galaxies at faint magnitudes in the steep case. When viewing the figures it is useful to note that fainter than $B_J = -17.0$ the entire contribution to the luminosity function is from galaxies with $z < 0.1$, where the relative errors in redshift are very high $\sigma_z/z \approx 1$. The uncertainty in deriving the luminosity function is dependent on the redshift error, σ_z. When we perform the simulations using the errors expected from higher quality photometric data the luminosity functions are recovered more accurately (see Fig 3 and Fig 4).

3. Application of the Technique

3.1. The Data

We construct a sample of galaxies with photometric redshifts from the photometric and spectroscopic survey data of Koo and Kron (Kron 1980, Koo 1986). These data consist of scans of photographic plates taken with the KPNO 4m. They cover the U, B_J, R_F and I_N passbands and are 50% complete at a magnitude limit of $B_J \sim 24$. Details of the photometric data can be found in (Bershady et al. 1994, and references therein), while the spectroscopic data are described in Munn et al. (1996).

Our current analysis considers only the high Galactic latitude field Selected Area 68

(SA68). In this paper we describe the technique, and we do not attempt to combine fields in order to minimize possible errors from zero point offsets amongst different plate material. From these data we define two samples of galaxies. We derive a $b_J < 22.5$ magnitude limited sample of galaxies detected in all four passbands from which we can determine photometric redshifts. At this magnitude limit the relative uncertainties in the photometric data were less than 0.5 magnitudes in each of the four passbands. From the photometric sample we derive a subset of those galaxies with high quality spectroscopic redshifts. We use these two data sets to compare the relative accuracy and robustness of our analysis. The photometric redshift sample consists of 772 galaxies and the spectroscopic redshift subset has 114 galaxies.

We use the techniques described in C95 to estimate the galaxy redshifts from their multicolor broadband photometry. We apply the fits from C95 of a second order polynomial to the four optical passbands and derive a relation between spectroscopic redshift and broadband photometry. The photometric redshift distribution of this sample is given in Fig 5. Comparing the spectroscopic and photometric redshifts we find that, at $b_J < 22.5$, the dispersion in the photometric redshift relation is Gaussian with $\sigma_z = 0.045$ slowly increasing towards fainter magnitudes. The correlation between the redshift dispersion and r_F magnitude is given in Eqn 1.

3.2. K-corrections

We calculate the K-correction for each galaxy from the spectral energy distributions (SEDs) of Kinney et al. (1996) . We choose these spectra over stellar synthesis models as they are derived from actual galaxy spectra. We selected five SEDs from the Kinney et al. (1996) data: those of an Elliptical, S0, Sb and two starburst galaxies with E(B-V) = 0.05 (S1) and E(B-V) = 0.70 (S6). These five SED's were chosen to encompass the expected

color distribution of our galaxies. The Elliptical and S0 templates have very similar colors, so they were averaged together, and the remaining four templates were used in calculating the K-corrections. Fig 6 shows the $b_J - r_F$ colors of these templates as a function of redshift.

For each galaxy in the photometric and spectroscopic samples we derive an estimate of its redshift. We determine the spectral type of a galaxy from its observed $b_J - r_F$ color. To improve the accuracy of our K-corrections we interpolate between the two galaxy templates with the closest $b_J - r_F$ colors. From the estimated redshift and spectral type of each galaxy we calculate the K-corrections in each of the four bandpasses. The distribution of apparent $b_J - r_F$ color of the photometric sample is shown in Fig 6.

3.3.   Completeness

There are two sources of incompleteness that we need to be concerned about. The first is incompleteness in the photometric sample, and the second arises from the estimation of redshifts. We believe the first source of incompleteness to be minimal. The sample was conservatively cut at $b_J = 22.5$, which is a full magnitude and a half below the 50% completeness level (Koo 1986). The second source of incompleteness has its origin in the fact that the redshift estimation procedure occasionally indicates a negative redshift. We interpret the negative redshifts as reflecting the error distribution function which extends below zero for the lowest redshift galaxies. 49 of the 772 galaxies (6.3%) had redshift estimates which were negative. These galaxies are essentially lost from our sample. We consider three methods of dealing with their omission. The first is to simply to ignore them, this is the 'minimal' luminosity function. This has the advantage that it makes no assumptions about the missing data; however, in this case our luminosity function is strictly a lower limit. The second method is to weight the luminosity function by the missing fraction. This is the 'weighted' luminosity function. This method makes the explicit

assumption that the distribution of the missing galaxies is identical to those whose redshifts we could estimate. This assumption is supported by the fact that the incompleteness fraction appears to be a weak function of apparent magnitude. If the incompleteness is only dependent on redshift, then the incompleteness will leave our luminosity function estimates unbiased. Yet we know from Eqn 1 that this is not entirely true. The third method takes advantage of our expectation that the source of incompleteness is low redshift galaxies and cuts the low redshift galaxies from the sample. This is the 'cut' luminosity function. The 'cut' luminosity function has the further advantage that it is in the low redshift regime where the error distribution in absolute magnitude is the broadest. We cut from the sample the contributions of galaxies with $z < 0.1$. Remember that in our scheme galaxies occupy a distribution in redshift. When we make a cut in redshift we only consider the fraction of each galaxy's probability distribution between the redshift limits.

3.4. Results

We have applied our luminosity function technique, including bias corrections, on the photometric redshift sample described above. We derive luminosity functions for the 'minimal', 'weighted' and 'cut' galaxy samples. In Fig 7 the 'minimal' luminosity function is shown bracketed by lines denoting a one sigma standard deviation. The luminosity function is represented as a curve rather than the usual points since our technique produces a continuous estimate of the luminosity function. Fig 7 also shows the number of galaxies as a function of absolute magnitude. This shows roughly how many galaxies are contributing to the luminosity function estimation in each magnitude interval.

The luminosity functions for each of the three samples are reasonably well fit by Schecter functions. Table 1 shows the best fit Schecter parameters for the three luminosity functions. For all of the samples, the parameters derived are consistent within one standard

deviation.

The values of M^* and Φ^* agree within the errors to the values derived by Loveday et al. (1992). Yet our luminosity functions appear to have a steeper faint end slope, even when the bias in Fig 3 is taken into account. The luminosity function of the CfA redshift survey also shows a considerable faint galaxy excess (Marzke et al. 1993). Keeping in mind that the photometric redshift estimation tends to smooth the luminosity function, we might expect a luminosity function with a strong faint end turn up, such as that of the CfA, to be better fit by a Schecter function with a steeper faint end slope. The large number of inferred faint galaxies also helps to reconcile the number counts and the galaxy redshift distribution (Gronwall and Koo 1995).

We apply a further test of our technique by comparing the results from the photometric sample to the luminosity function derived from a sub-sample of 114 galaxies with high quality spectroscopic redshifts. These galaxies are from one of a number of fields whose luminosity function is estimated in a upcoming paper by Koo et al. (1996) . In order to fairly compare the spectroscopic sample with the deeper photometric sample we have cut back the sample to $b_J = 20.0$. The results are shown in Fig 8. The luminosity derived from the photometric data is shown by a continuous curve, and that from the spectroscopic redshift sample by the points. It is readily apparent that the two luminosity functions agree very well.

4. Conclusions

We have presented a new technique for the determination of luminosity functions from photometric redshift samples. This technique considers the statistical scatter in the photometric redshift procedure and integrates over the redshift probability distribution for

each galaxy. The result is a continuous estimate of the luminosity function. This procedure can easily be generalized for use when the errors in any of the parameters are large. The large cost benefit in obtaining photometric vs. spectroscopic redshifts indicates that this technique can be an important tool in the multivariate analysis of galaxy properties.

In conclusion we would like to make the following points regarding our technique and its application:

(1) The results from our technique are in good agreement with those from deep spectroscopic surveys. Our results do seem to indicate a very large number of faint galaxies. However, at the faint end of the luminosity function there appears to be little agreement among different groups (Loveday et al. 1992,Marzke et al. 1994,Lilly et al. 1995).

(2) Our technique will do better as we go to fainter samples and hence higher redshift galaxies. Low redshift galaxies are difficult to deal with for two reasons. The first is that the size of the error distribution in absolute magnitude diverges as you approach $z = 0$. The second reason is that the photometric redshift estimation procedure occasionally indicates a negative value for some, presumably, low redshift galaxies. These galaxies create incompleteness in our sample.

(3) Our technique is currently the most practical method for pursuing a multivariate study of the galaxy luminosity function. It is certainly informative to split the luminosity function in the two dimensions of redshift and spectral type (Lilly et al. 1995). The efficiency of the photometric redshift estimation makes it conceivable to split the luminosity function in three or even four dimensions.

REFERENCES

Baum, W.A., Problems of Extragalactic Research, IAU Symposium No.15., 390.

Bershady, M.A. et al., 1994, AJ, 108, 870.

Caditz, D.C., & Petrosian, V., 1993, ApJ, 416, 450.

Choloniewski, J., 1986, MNRAS, 223, 1.

Connolly, A.J., Csabai,I., Szalay, A.S.,Koo, D.C., Kron, R.C., Munn, J.A., 1995, AJ, 110, 2655.

deLapparent, V., Geller, M.J., Huchra, J.P., 1989, ApJ, 343, 1.

Eddington, A.S., 1940, MNRAS, 100, 354.

Efstathiou, G., Ellis, R.S., Peterson, B.A., 1988, MNRAS, 232, 431.

Ferguson, H.C., McGaugh, S.S., 1995,ApJ, 440, 470.

Kinney A.L., Calzetti, D., Bohlin, R.C., McQuade, K., & Storchi–Bergmann, T., 1996, ApJ, (in press).

Koo, D.C., 1985, AJ, 90, 418.

Koo, D.C., 1986, ApJ, 311, 651.

Koo, D.C., Gronwall, C., Bruzual, A.G.,1993, ApJ, 415, L21.

Koo, D.C., et al., 1996, in preperation.

Kron, R.G., 1980, ApJS, 43, 305.

Lilly, S.J., Tresse, L., Hammer, F., Crampton, D., LeFevre,O., 1995, ApJ, 455, 108.

Loveday, J., Peterson, B.A., Efstathiou,G., & Maddox, S.J., 1992, ApJ, 390, 338.

Lynden–Bell, D., 1971, MNRAS, 155, 95.

Malmquist, K.G., 1920, Medd. Lund. Astr. Obs., Series 2., No. 22.

Marzke,R.O.,Huchra,J.P.& Geller,M.J., 1994, ApJ, 428, 43.

Munn, J.A., Koo, D.C., Kron, R.G.,Majewski, S.R., Bershady, M.A., Smetanka, J.J., 1996, ApJS, (in press)

Petrosian, V., 1992 Statistical Challenges of Modern Astronomy,eds. Feigelson, E.D. & Babu, G.J., Springer–Verlang, p173.

Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.T., 1986, Numerical Recipes: The Art of Scientific Computing, Cambridge University Press. Statistical Challenges of Modern Astronomy,eds. Feigelson, E.D. & Babu, G.J., Springer–Verlang, p173.

Schecter, P., 1976, ApJ, 203, 297.

Schmidt,M. 1968, ApJ, 151, 393.

Fig. 1.— This figure illustrates the mathematical quantities defined in the derivation of the C-method. We are only able to observe the galaxies below the limiting magnitude line. In this apparent vs. absolute magnitude plot, a 45 degree line represents, neglecting differential K-corrections, a line of constant distance. Ψ is the true number of galaxies brighter than M. X(M) is the observed number of galaxies brighter than M. In general $\frac{d\Psi}{\Psi} > \frac{dX}{X}$ since it is generally harder to observe fainter galaxies. We define the quantity C so that $\frac{d\Psi}{\Psi} = \frac{dX}{C}$. C is the number of galaxies in the shaded region of the figure. Each galaxy in this region could have been observed if their absolute magnitude was M'.

Fig. 2.— This figure illustrates the the same quantities as Fig 1, but now each galaxy is represented as a probability distribution in absolute magnitude. This probability distribution results from the error associated with the photometric redshift estimation. The fraction of a galaxy which contributes to the quantity C is the middle shade of grey.

Fig. 3.— The finite error in the redshift estimate results in a bias in the derived Schecter parameters M^* and α (similar to a Malmquist bias). The figure on the left is for the errors in redshift given by Eqn 1. The figure on the right is for errors in redshift one-third of that, which represent the highest accuracy obtainable through the photometric redshift method. The dotted arrow shows the deviation from the true values of M^* and α when no bias correction is applied. The solid line shows the deviation in the parameters after we have applied our bias correction. The 90% confidence contours are plotted for the simulations with the bias correction. These results were determines for samples of roughly 800 galaxies.

Fig. 4.— Results from the simulations. The two figures on the top had input luminosity functions of $\Phi^* = 2 \times 10^{-3}$, $M^* = -21.0$, $\alpha = -1.5$ and the two on the bottom had input luminosity functions of $\Phi^* = 3 \times 10^{-3}$, $M^* = -21.0$, $\alpha = -1.0$. The figures on the left had redshift errors given by equation 1, while the figures on the right are for errors one third of that, (which represent the highest accuracy obtainable through the photometric method).

The solid lines are the input luminosity functions and the three dashed lines represent the recovered luminosity function and the one sigma spread in the estimates.

Fig. 5.— The redshift distribution for SA68 derived from the photometric redshift sample

Fig. 6.— The colors of the galaxies in our sample overplotted with the four template spectra from Kinney et al.. K–corrections are interpolated using the two closest template spectra.

Fig. 7.— The resulting 'minimal' luminosity function derived from the photometric redshift sample. The line is bracketed by its one sigma errors. The 'cut' luminosity function turns out to be virtually identical except it contains no estimate fainter that $B_J = -17$. The distribution in absolute magnitude is also plotted to give an understanding of how many galaxies contribute at each magnitude interval.

Fig. 8.— A comparison of the luminosity function derived from the photometric redshift method (solid line) with that calculated using the spectroscopic redshifts (points with error bars). To facilitate a direct comparison the photometric sample was cut to similar limits, $b_J = 20$, as the spectroscopic sub-sample. The two luminosity functions are in good agreement.

Table 1.   Schecter Luminosity Functions Derived from the Phometric Redshift Sample

Type	$\Phi^* - 5[h_{50}^3 Mpc^{-3}]$	$M^* - 5\log h_{50}$	α
Minimal	$2.64 \pm 0.85 \times 10^{-3}$	-21.17 ± 0.16	-1.48 ± 0.12
Weighted	$2.80 \pm 0.90 \times 10^{-3}$	-21.17 ± 0.16	-1.48 ± 0.12
Cut	$2.80 \pm 0.77 \times 10^{-3}$	-21.13 ± 0.15	-1.46 ± 0.12